SKETCH OUTLINE:
PLAYING WITH EVENS AND ODDS (WW27 and WW30)

Folk in the mathematics community are aware of the following astounding fact:

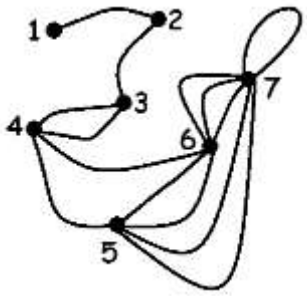
> *The number of people, living or deceased, who have taken part in an odd number of handshakes up to this very moment in time, is even.*

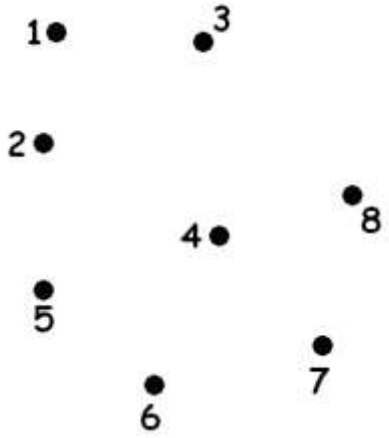
How can one possibly verify such a claim?

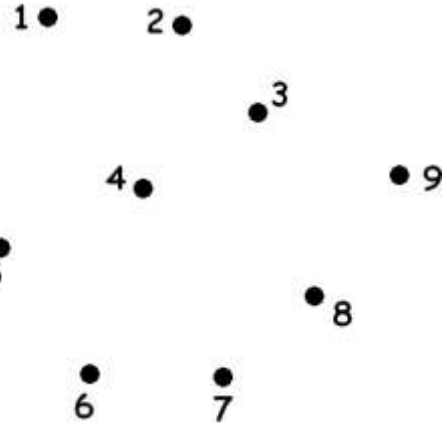
> Fun exercise: With an odd number of people in a room, you and some even number of friends, have each person try to take part in an odd number of handshakes. What happens?

In his book WITHOUT WORDS (Tarquin, 2015), James Tanton provides two wordless puzzles that develop the thinking needed to explain this handshake fact. Both puzzles ask you to draw lines between pairs of dots (or from a dot to itself) so that the indicated number of lines emanate for each dot. Which diagrams have solutions and which do not? (What is the obstruction?)

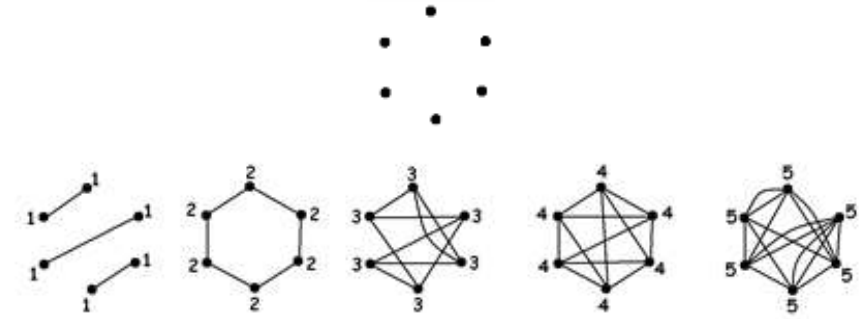
PUZZLE 1:

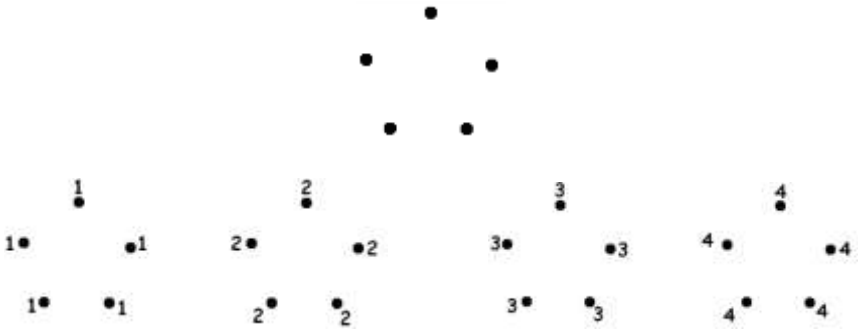




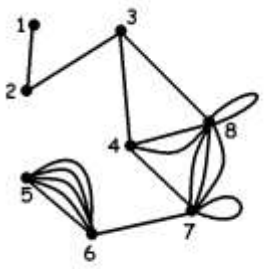


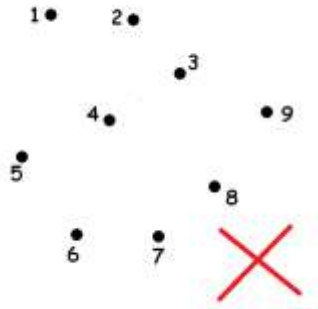
PUZZLE 2:





SOLUTION:

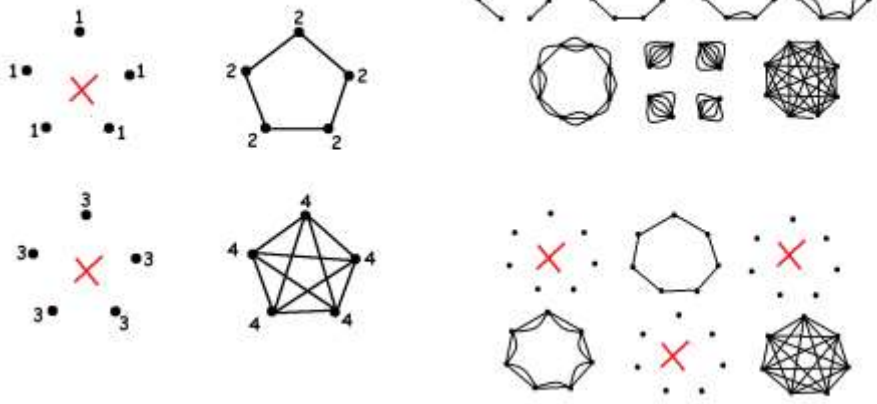




This puzzle wants us to draw edges between dots so that the count of edges emanating from a given dot matches the number at that dot. The first puzzle can be solved (in many different ways), but the second can not. Here's why.

> Each line has two "ends" and so, in any completed diagram, the total number of line ends must be even. But the second diagram wants $1+2+3+4+5+6+7+8+9=45$ line ends. It cannot be solved.

Challenge: For which N can the puzzle with N dots numbered $1, 2, 3, \ldots, N$ be solved?



Again, in any diagram the total number of line ends must be even. Any diagram that requires an odd total number of line ends (it has an odd number of cells requiring an odd number of lines emanating from it) is impossible to complete.

For handshakes … if we represent each person as a dot on diagram and each handshake between two people as a line between the matching pair of dots, then the total count of line ends, that is, the sum of all the counts of handshakes folk say they took part in, must be even. There cannot be an odd number of people, then, claiming to have taken part in an odd number of shakes.