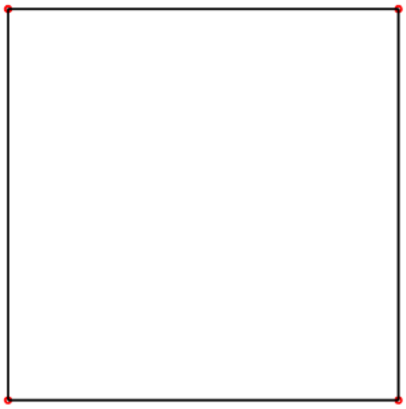
Drawing Pursuit Curves

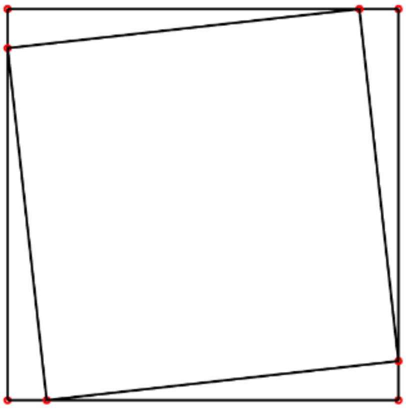
When predators follow simple strategies for hunting their prey, the paths they move along are called **pursuit curves**. The simplest strategy is “move toward your prey”! Depending on how the prey moves and how fast the predator moves, this strategy can produce some beautiful curves. When there are three or more identical animals and they are all both predator and prey, the animals move along pursuit curves called logarithmic spirals.

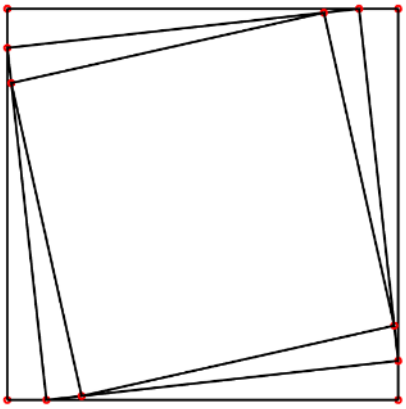
Four Mice in a Room

Suppose four mice are in a square room, each in its own corner. Each mouse follows an identical strategy: run at a constant speed towards their nearest anticlockwise neighbour. All four mice start moving at the same time, and at the same speed. In real life, mice constantly adjust their direction in order to move directly towards their prey, but it is not easy to find the shape of those paths mathematically. However, we can draw an approximation of the paths by assuming that the mice have a bit of a lag: they run in a series of straight lines towards where their prey *was* at the time they start each straight line run. We can do this by following the steps below.

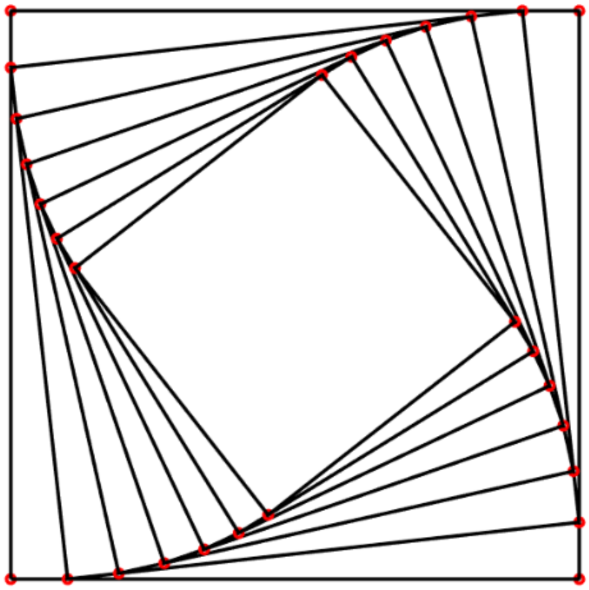
1. Draw four dots to indicate where the four mice are at the start – in this case, the corners of a square room. Then draw straight lines to connect each dot to its nearest anticlockwise neighbour, producing a square with one dot at each corner. (Use our template if you prefer.)
2. Next, each mouse moves 10% of the distance along the lines drawn in Step 1, towards where their nearest neighbour was at the end of that step. Measure and mark their new positions with dots and join the four new dots with straight lines, as in the middle diagram below.
3. The mice again move 10% of the distance to the position of their nearest anticlockwise neighbour at the end of the previous step, such that the new mouse positions are 10% of the way along each of the straight lines drawn in the previous step. Draw dots at these positions as in the right hand diagram below, and join the dots with straight lines.

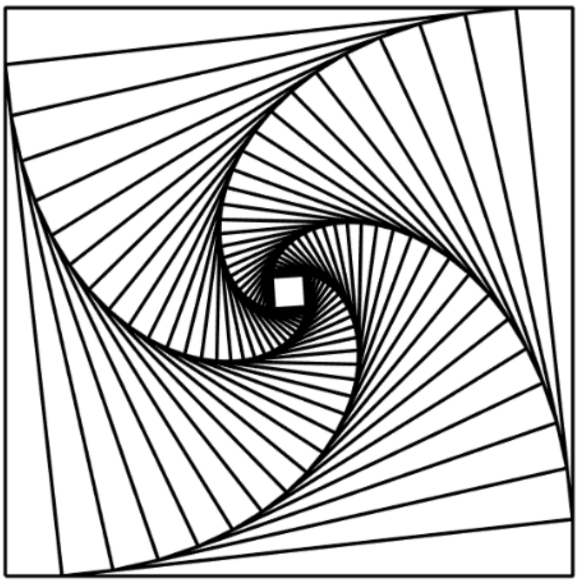






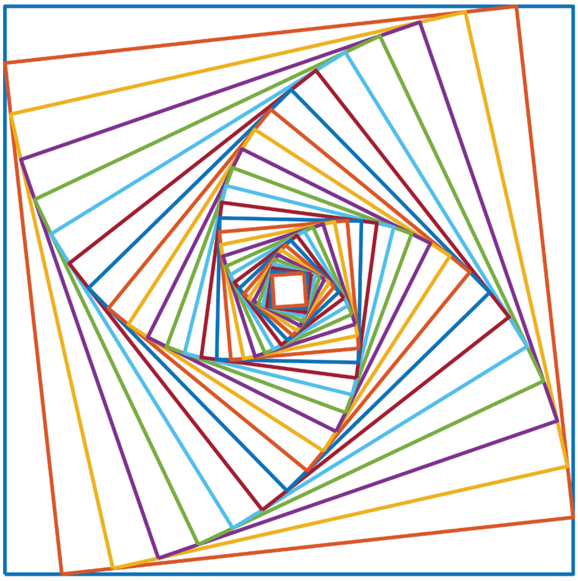
4. Repeat Step 3 until you have drawn as many squares as you want. Notice that the mice cover a smaller distance with each step, so you have to measure and work out 10% of the distance at each step. You will draw a sequence of smaller and smaller squares, each rotated anticlockwise with respect to the previous one. On the left below is what the drawing looks like after seven steps.
5. After you have drawn as many squares as you want, you will have produced a close approximation of four logarithmic spirals traced out by the corners of the squares, each spiral corresponding to the path of a mouse. In the diagram on the right below, we have stopped after 30 steps (the dots have been left off to make it clearer).





Further Explorations

You can also colour in the many triangles created by the black and white spirals seen above, as in our *Curves of Pursuit* colouring handout, or you can draw the squares with coloured lines as below.



If you like making these spirals, you can try to vary the rules we followed above. For example, what if there are three mice in a triangular room, or five in a pentagonal one? What if they can move different distances compared to each other in each step, or can change the distance they move between steps? What if instead of chasing their nearest neighbour, they chase another mouse? Can you think of other things to change? Can you think of some interesting questions to ask about the curves you produce?