Mathematical Balloon Twisting

Balloon twisting is a well-established art form, but mathematical models have gained prominence in the repertoire of the balloon-twisting world only recently. This booklet includes instructions for many polyhedra, including all five platonic solids. There are also instructions for four interesting polylinks, here called ‘tangles.’ These models are efficient, in the mathematical sense that they traverse the edges with the least number of balloons possible.

Balloons are a fun and inexpensive medium through which to enjoy mathematical thinking. Experience shows that students’ openness to balloons makes for engaging applications in math education.

More information about mathematical balloon twisting can be found on my website at: vihart.com

Happy twisting!

1

Practical Tips

Balloons come in many sizes and shapes, but here we are concerned with two different kinds of long skinny balloon: 160s, which are 1” x 60”, and 260s, which are 2” by 60”. They’re both easy to find and order online. I recommend staying away from metallic and pearl colors, which aren’t as stretchy. You’ll also need a pump, because these kinds of balloons are nearly impossible to inflate without one.

Whenever you make a twist in a balloon, the air will try to expand into the end, so be sure to leave enough of the balloon uninflated that the air has room to expand for each twist you make. For the same reason, start twisting at the inflated end, moving towards the tail.

![Image of balloons twisted into shapes](image)

Most models require that there’s still enough tail left when you’re done twisting that you can connect it to something else, so be aware of that as well!

When making something out of many balloons, it is useful to inflate all the balloons before you start twisting, so that you can be sure you’re inflating them to the same length. You can also leave one untwisted balloon inflated to the right length to serve as a guide, as you inflate and twist others.

One more trick: if you ever find yourself wanting to pass the end of the balloon through a space too tight for it, you can squeeze the uninflated end through, and then pass the air through a tight choke. You can get it started by putting the end in your mouth. Don’t inhale with your lungs, but suck the air into your mouth much as you would suck on a straw. Don’t choke, either.

![Image of balloons twisted into shapes](image)
Octahedron

The octahedron can be twisted nicely out of one balloon. A 160 is easiest, but a 260 works too.

1. Twist off five sections, bunching them up in your hand so that they don’t untwist. Connect the first and last twists you made, to create a square.

2. Twist two more sections, and connect to the opposite side of the square.

3. Twist another section and connect it to the first end of the balloon.

4. Twist off the last four sections, connecting each to the closest degree-2 vertex, and then join the two ends.

Note that the graph of the octahedron’s edges has an Eulerian cycle, meaning you can traverse all edges and end where you started, so you can make it out of one balloon. This is true for any polyhedron where an even number of edges meet at each vertex.

Triangular Dipyramid

The triangular dipyramid can also be twisted out of one balloon. Either 160s or 260s work.

1. Twist off four sections, and connect the first and last twists you make, to create a triangle.

2. Twist off three more sections. Connect the second twist with one vertex of the triangle, and the third with the first end of the balloon.

3. Make one final twist, connect it to the other vertex of that first triangle, and then attach the end of the balloon to the remaining degree-2 vertex.

Notice that the triangular dipyramid has an Eulerian path, not a complete cycle. The path starts at one odd-degree vertex and ends at the other.
Tetrahedron

The tetrahedron is the simplest polyhedron, but it requires two balloons to make. 260s are preferable, twisted into three sections each.

The tetrahedron has four vertices, all of degree three. A balloon must end at every odd-degree vertex. Because each balloon contributes two ends, and \( \frac{4}{2} = 2 \), two balloons are necessary.

Cube

The cube requires 4 balloons, preferably 260s. Each balloon is twisted into three sections.

There are two ways to put these four parts together, both of which have cyclic symmetry. One has four Zs chasing each other in a circle, and the other has four Cs.

Dodecahedron

The dodecahedron requires 10 balloons, twisted into three sections each. 260s are best.

Similarly to the cube, it can be made with either Cs or Zs in cyclic symmetry. First connect each five-piece half, then put the two together. The halves must be mirrors of each other (Zs and Ss, in the example below).
**Icosahedron**

1. Twist a 260 into 5 equal sections, and create the form of two equilateral triangles sharing an edge.

2. Create 6 of these modules, possibly two each of three different colors.

3. Use the nubs at the ends of each balloon (located at the vertices where 3 edges meet) to connect the unit to the ends of another unit (where two edges meet). Each vertex of the finished icosahedron will be of degree 5, and the center edges of each module will lie on the faces of a cube.

*This isn’t the only symmetric way to put six balloons together into an icosahedron. I’ll leave the others as a puzzle!*

**Snub Cube**

The same module used to make the icosahedron can also make the snub cube. You’ll need 12. Four each of three colors works nicely.

**Snub Dodecahedron**

Or, if you have 30 balloons and lots of patience (or friends), make the snub dodecahedron! It can be made using 6 each of 5 colors, or there is also a nice arrangement using 5 each of 6 colors.
4-Simplex

The four-dimensional analog of the tetrahedron can be made from one balloon. Both 160s and 260s work. This model has two different lengths of edges. Getting the proportions right can be tricky.

1. Twist two short sections, a long, and another short. Connect the first and last twists. This connection will be the center vertex.

2+3. Twist off the fourth and final short section, then continue twisting off long sections to cover all the outside edges.

Each of the five vertices connect to the four others, making a complete graph. If we had higher-dimensional balloons, you could make all the edges the same length.

Tangles

Tangles are made up of polygons linked in a symmetric way. They can be a puzzle to put together, but look very graceful when finished.

Getting the polygons to fit snuggly together is important. If the edges are too thin, the structure falls apart, and if they’re too thick, the structure will be warped. The perfect lengths can be made by first putting the tangle together with conservatively thin edges, and then grabbing each corner and twisting off a bit. Below, and on the front cover, is the tangle of 6 pentagons.

Each pentagon is linked with the five others in a symmetric manner.

The convex hull is an icosahedron.

Borromean Rings

The Borromean rings are a tangle of 3 rectangles with the property that, though all three are locked, popping any one balloon will leave the other two unlinked.

Both 160s and 260s are too skinny to get a snug fit. Either don’t inflate the full length of balloon, or use 260s and then make a twist at each corner.
Tangle of Six Squares

This tangle is made from three pairs of squares. 260s are about the perfect length, but 160s look good if you twist the corner of each square as described earlier. It’s nice in three colors, with parallel squares being the same color.

Weaving it is the hardest part, but steps 2 and 3 of the diagram show bands that should make it clear how to weave.

The six squares lie on the faces of a cube. Each one links with four others, but is unlinked to the one it lies parallel to.

Tangle of Four Triangles

The four triangles are each parallel to the planes of the four faces of a tetrahedron, but all four planes intersect in the center. Each triangle is linked with every other one. 260s work best, and a twist at each vertex is needed, unless they are not fully inflated.

Five Intersecting Tetrahedra

Though a tetrahedron can be made out of 2 balloons, to get the proportions right I made each out of 3 260s. Putting this together is a real challenge-- it helps to have a model as a guide.
References

For a longer introduction see:

For ideas about using balloons in K-12 education:

For more about the mathematics and NP-hardness of certain balloon design problems, see:

Search YouTube for “balloon polyhedra” to find many instructional videos. There is also more information on my website: vihart.com

Photos by: Erik Demaine and Iuliu Vasilescu

24